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2301. Place the quadrilateral with PR along the x axis, and QS going through the origin. Then, using the area formula $A_\triangle = \frac{1}{2}bh$, consider the coordinates of the vertices Q and S.

2302. Write each side as 2^*.

2303. Calculate the square side length c^2 in two different ways, and equate them.

2304. Find the probability that the five cards dealt are $3, 4, 5, 6, 7$ in that order, and then multiply by the number of orders.

2305. There is no guarantee that y and z are correlated. Describe (there's no need to detail it specifically, although you could) how a counterexample might come about.

2306. Find the equations of tangents at points $x = a$ and $x = b$, in the form $y = mx + c$. Both m and c will be in terms of a or b. Then solve simultaneously.

2307. Consider the ellipse as a stretched circle.

2308. Solve the inequality, considering the solution set as an interval of successful outcomes. The domain $[0, 1]$ is the possibility space.

2309. Consider the sign changes of each of the factors as x passes through 0: the question is whether the total number of sign changes is odd or even.

2310. You don't need to multiply all three denominators together. Use their LCM. The third denominator is a factor of the first, so the product of the first two will serve.

2311. Consider instead the integral of $\sec^2 x$, which is $\tan x$. Then convert the result by switching cos and sin. You'll need an extra minus sign.

2312. (a) Each roll should have three branches.
   (b) Sum two probabilities.
   (c) The tree diagram continues infinitely beyond the first two rolls.
   (d) Sum an infinite geometric series. Find the first term a and common ratio r explicitly and use $S_\infty = \frac{a}{1-r}$.
   (e) Consider the possibility space $\{1, 2, 3, 6\}$.

2313. Consider the difference between the sets of points that satisfy the two inequalities.

2314. Use the fact that $\log_a b = \log_{a^n} b^n$, and then the standard log rules.

2315. Calculate the vertical height of the pyramid using Pythagoras. Then use similar triangles.

2316. Translate into a fact about signed areas between the line and the y axis. Or, if you'd find it easier to work with x, replace all the y's with x's first and use signed areas between the line and the x axis; just remember to convert back at the end.

2317. (a) Set vertical velocity to zero and solve for t.
   (b) Consider the fact that the projectiles all have constant horizontal speed. This means that the shortest distance between them occurs when they are at the same height as each other.

2318. Set up an identity with the information given, and equate coefficients.

2319. To find the area of the triangle, you can use the cosine rule and $\frac{1}{2}ab\sin C$, or alternatively Heron's formula.

2320. Assume that the kings cannot be placed on the same square. There are three different locations for the first king: corner, side, middle. Deal with these three cases separately.

2321. You can ignore the $+k$, since it simply translates the entire problem in the y direction.

2322. Draw a triangle of forces, and use the cosine rule.

2323. In each case, set the input to y and the output to x, and then rearrange to make y the subject. In (c), you'll need to use g as the inverse of g^{-1}.

2324. (a) Differentiate implicitly and rearrange to make $\frac{dy}{dx}$ the subject.
   (b) Find the coordinates of the intercepts, then find the equations of the tangents, then solve simultaneously.

2325. Count outcomes in the possibility space.

2326. Turn the AP information into a single equation by equating the differences, and do similarly for the GP information. Solve simultaneously.

2327. Consider the formula for the area of a trapezium, or else use a definite integral. You'll need to use $v = u + at$ for the final velocity (which is true by definition of acceleration).

2328. This is a quadratic in $\sin^{-1} x$. Factorise it, and then consider the range of the arcsin function.

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2329. This isn't necessarily true, though it sometimes is. You need to consider the form of the alternative hypothesis.

2330. These statements are the negations of those in the factor theorem. Think about the direction of the implication there.

2331. Establish algebraically whether or not the second derivative changes sign at $x = 0$.

2332. Consider the translational and rotational effects separately. Firstly, show that no combination of magnitudes yields zero resultant force. Secondly, take moments around the axis of symmetry.

2333. Show that the intersections are independent of k, and that the y difference between them is too. You don't need to calculate the integral explicitly.

2334. (a) Reciprocate and square both sides.
(b) Square and simplify $2 + \sqrt{3}$. Also rationalise the denominator of the result in (a); show that these are the same.

2335. This is the same as solving a pair of simultaneous equations in a and b.

2336. The first sentence is correct; the second is not. The question is which two objects have potential relative motion between them. To visualise this, replace the running track with an ice rink.

2337. The first sentence is true, the other two are not.

2338. Multiply up by $(x^4 + x^2)$ as if you were trying to find A, B, C. Then equate coefficients. Carry on through the method until you find a contradiction.

2339. Consider the three-way product as $((uv)w)'$, and use the two-way product rule twice.

2340. Choose the chairperson first, then the secretaries from the group remaining.

2341. Sketch the two circles, and compare the distance between the two centres to the radii.

2342. (a) Find the acceleration of the system by treating the two masses on the slope as a single object. Then consider only the lower one.
(b) Note that the acceleration will not change from part (a).

2343. This is a geometric progression. Find an ordinal formula, substitute it into the boundary equation of the inequality, and solve using logarithms.

2344. Use the compound-angle formulae.

2345. The interpretation is the fact that, for a circle, tangent and radius are perpendicular.

2346. Multiply up by the denominators, and rearrange to a quadratic in 2^x. Solve for 2^x using the quadratic formula, before taking logs. Consider the signs carefully.

2347. Give the vertices position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$. Then find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{SR}$.

2348. (a) Proceed algebraically, or (easier) consider the distance between the centres of the circles.
(b) A sketch will help, as ever!

2349. Find the first derivative of each, in terms of f' and/or f. Then substitute in a root $x = \alpha$, and use the fact that $\mathrm{f}(\alpha) = 0$.

2350. By symmetry, the x coordinate is the mean of a and b. Substitute this back in for the y coordinate.

2351. Use the formula $\binom{n}{r} \equiv {}^n\mathrm{C}_r \equiv \dfrac{n!}{r!(n-r)!}$.

2352. Use the second Pythagorean trig identity. You don't need to do any differentiation (although that would also work).

2353. This is, in a sense, obvious. Often, the easiest way to explain an obvious result is by being formal, generally in algebra; then there's no doubt about what you're saying.

2354. Square root both sides, and sketch both resulting curves. The standard curve you need to refer to is $y = \frac{1}{x^2}$.

2355. (a) Carry out both integrals.
(b) Take the limit.
(c) Solve a quadratic in y.

2356. Work out whether the minimum of the graph $y = \mathrm{f}(x)$ lies within the given domain. Then sketch the function over the given domain.

2357. Solve to find the intersections of the curves, which will be a surd expression. Set up a single definite integral, carry it out and simplify the surds.

2358. Integrate once to find the velocity, including a $+c$. Use the average velocity information to find this $+c$. Then integrate again, justifying your choice of $+d$ with the meaning of the word *displacement* (as opposed to *position*).

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2359. Substitute to form the equation for intersections, and set the discriminant to zero.

——— Alternative Method ———

Use circle geometry: the fact that the tangent and radius are perpendicular.

2360. The numbers of dots on opposite faces of a die add up to 7.

2361. A "linear approximation" is another way of saying "tangent line".

2362. Substitute the conditions in to get two equations linking A and k. Solve by dividing one equation by the other.

——— Alternative Method ———

Consider the fact that, in exponential growth, the time taken for the population to double in size in constant.

2363. Write the sum out longhand, simplify, and then solve the resulting quadratic.

2364. This involves showing that the three lines form an equilateral triangle. This is most easily done by considering symmetry in the line $y = x$, but can also be done by explicitly finding the points.

2365. One piece can be placed without loss of generality. Then consider the other three. Each tile has four possible orientations.

2366. Equate the two instances of the common ratio, and solve to find c. Then work backwards to get the first term.

2367. (a) Differentiate $\sec x$ either by quoting a result or using the chain/quotient rule.
(b) Using a trig identity, rearrange the derivative in terms of $\sec x$. Consider the sign when you take the square root.

2368. Show that the relationship between infinitesimal changes in x and u is given by $dx = \frac{1}{2}du$. Enact the substitution, remembering to change the limits from x values to their respective u values. Once you've done the substitution, split the fraction up, writing it in the form $au^m + bu^n$. Then integrate the terms individually.

2369. Set the first derivative to zero to find SPs. Use this to find the range of $x \mapsto \cos^2 x + \cos x$. Convert this into a range for the reciprocal.

2370. Two of these are necessarily tangential.

2371. You don't need to draw a full possibility space for this one, just list and consider the outcomes which sum to 7.

2372. Use the fact that $\sin\theta$ and $\cos\theta$ are reflections of each other in $\theta = \frac{\pi}{4}$.

2373. (a) Show that $\mathrm{f}^2(x) \equiv x$.
(b) Set up the identity $\mathrm{f}^2(x) \equiv x$ with general a and b. Simplify and equate coefficients.

2374. Don't multiply out! Take out a common factor.

2375. Consider the values separately. So, assume that the lower block doesn't slip over the table, and model the upper block. Then assume that the two blocks don't slip against each other, and model both as a single object.

2376. Look for common factors.

2377. Use the standard formulae for area and arc length, calling the radius r, and solve. Consider the sign of your answer(s).

2378. (a) Use the negative reciprocal.
(b) Solve simultaneously.
(c) Consider the fact that Q is the square of the distance from the origin.

2379. You can square, then rearrange, then square again to reach a quartic, which can then be solved using a calculator's polynomial solver, or by numerical methods.

2380. Show that the cubic equation $ax^3 - bx = 0$ doesn't have the correct number of roots.

2381. You can do this without calculating p, q. Simply reflect the equation in $x = 0$.

2382. These are both true.

2383. Using implicit differentiation, enact $\frac{d}{dx}$ and then rearrange for $\frac{dy}{dx}$. Then reciprocate.

2384. Integrate the jerk twice, first indefinitely, and then definitely. Make sure you justify the choice of $+c$ in the formula for the acceleration.

2385. (a) Set $\mathrm{f}(x) \equiv \mathrm{f}^{-1}(x)$ as an identity, and equate coefficients.
(b) Proceed either algebraically or visually.

2386. (a) Use circle geometry, not algebra.

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(b) The area common to both circles is twice the area of a segment. Find the area of a sector and subtract the area of a triangle.

2387. Use $p = \frac{\text{successful}}{\text{total}}$.

2388. (a) Note that the curve $y = (3x-1)^3$ has a triple root at $x = \frac{1}{3}$.
(b) Consider the reverse chain rule.

2389. A problem on a clock is easier in degrees.

2390. For a counterexample, you could use a triangle with vertices at $(0,0), (2,0), (0,1)$.

2391. Show this by direct argument: simply set up two general rational functions, add them, and put the sum into a suitable form.

2392. (a) This is a similar process to completing the square. It is the coefficient of the x^2 term that dictates the value of k.
(b) Take out a factor of $(x-k)^2$.

2393. Combine the vectors, and use 3D Pythagoras.

2394. (a) Consider the value of x for which the fraction is undefined.
(b) Use the quotient rule to find the SP.

2395. (a) Use a tree diagram, conditioned on the choice of drawer.
(b) Restrict the possibility space to two of the branches.

2396. Set up $a(k+1)^3 + b(k+1)^2 + c(k+1) + d$. Show that $a = 1$ by comparing coefficients of x^3, then consider the coefficients of x^2 and so on.

2397. This is a positive quartic. For the details of the shape, consider the multiplicity of each root.

2398. For positive x, these are obviously true. Consider negative x.

2399. As it stands, calculus would be a bit complicated. Begin by letting $X = 2x + 5$.

2400. If in doubt, you can always write a sigma-notation sum out longhand.

——— End of 24th Hundred ———

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